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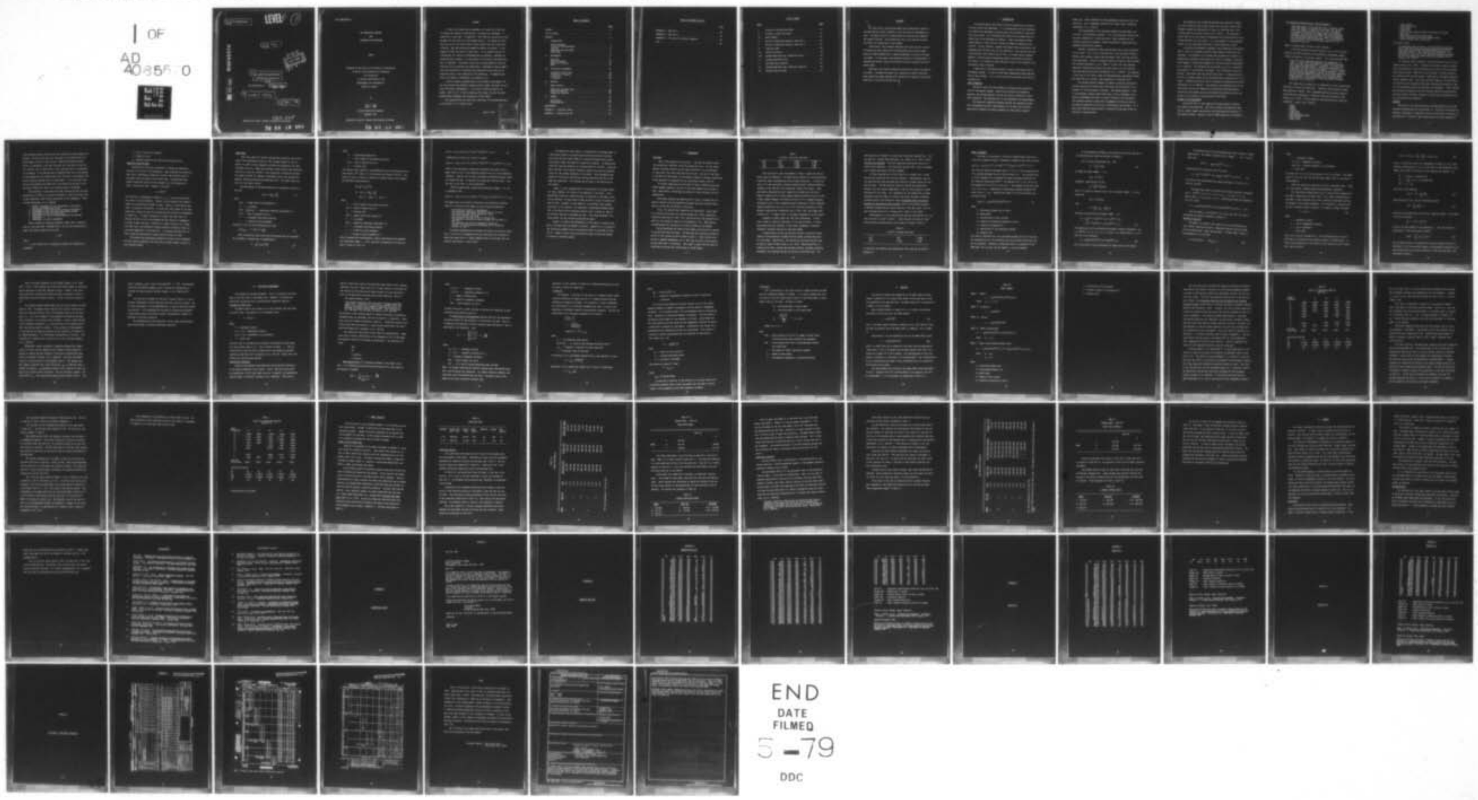

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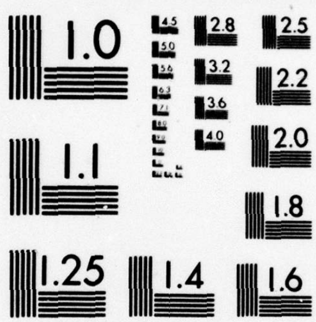

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THESIS

AFIT/GOR/SM/78D-8

10 John A. Long
Capt USAF

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THE PRODUCTION FUNCTION

AND

AIRFRAME COST ESTIMATION

THESIS

Presented to the Faculty of the School of Engineering

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by

John A. Long
Capt USAF

Graduate Operations Research

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PREFACE

To some, this thesis may be regarded as an exercise in futility. To capture the essence of uncertainty is to assume all knowledge. A prediction is just that - a prediction. Only time can lay bare the truth. No model for the future will ever become reality. All models are false. They are not the real world and will never predict the real world with certainty. Why then should one attempt to predict the future? To not attempt to model the future is fools play. In our complex society all alternatives, all sources of knowledge must be explored. Although the prediction may be wrong, it is the process of arriving at the prediction that is important. The process allows one an understanding of the complexities of the variables and interactions. Knowledge of these complexities and interactions illuminates the alternatives and allows one to shape the future in the direction of the prediction. In combating the future, any weapon is preferable to no weapon at all.

I wish to express my appreciation to my adviser, Keith Womer, for his kind patience and brotherly guidance and my reader, Jim Dunn, for his time and careful consideration. I also wish to thank my typist, Eve Vaught. Most of all, I wish to thank my wife, Nancy, for her love and understanding throughout the preparation of this thesis.

The responsibility for any errors, omissions, or misrepresentations in this thesis is, of course, mine.

John A. Long

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ABSTRACT

In recent years, men and governments have become keenly aware of the huge capital outlays necessary in the acquiring of new weapons systems. Increased burden on limited capital has required more complete and careful planning. This planning has led to the need for accurate and timely cost predictions of new systems.

Historically, the variables affecting the future cost of aircraft airframes have been proven to be airframe weight and aircraft speed. These are often combined with a learning hypothesis to form an airframe cost model. In this paper, the production function of microeconomics is combined with weight, speed, and learning to form a nonlinear cost estimation model.

Nonlinear least squares regression analysis was used in evaluating this model. Although the results are inconclusive, based on the data used, weight and speed combined with learning still appear to be the best predictors of aircraft airframe cost.

I. INTRODUCTION

The accounting for and control of future expenditures are matters of great concern and importance. It is becoming more and more important for industry and government to obtain good cost estimates of future products and systems. Cost estimation and prediction are nowhere more evidenced than in the Department of Defense. Today's peacetime military establishment is faced with ever increasing budgetary demands and constraints. Rising inflation, coupled with greater future uncertainty, has compounded the difficult task of cost prediction. Cost overruns and cancelled projects have become a way of life. For these reasons, greater emphasis is being placed on prediction models and predictive accuracy. This thesis is an attempt to shed additional light on this essential, yet frustrating, area of concern.

A major part of the defense budget is spent on the acquisition of new weapons systems. In the Air Force these expenditures often take the form of new airframes. It is in this area that this thesis will have its concentration.

THREE APPROACHES

Basically, there are three methods of estimating the acquisition costs of new weapons systems. These are the industrial engineering approach, the analogy approach, and the statistical estimation or parametric approach. Each approach has its own advantages and disadvantages.

The industrial engineering approach defines the engineering tasks, tool requirements and labor requirements at a low task level and then applies work standards to obtain estimates for labor hours, material

usage, etc. These estimates are then aggregated to provide a total program cost. This is sometimes referred to as "grass roots" estimating (Batchelder, et al, 1969:2).

The disadvantages of this approach outweigh the advantages, particularly in the aerospace industry. First, the aerospace industry does not apply standards on a wide scale basis. Many standards are non-existent and must be estimated. Rapid technological change makes many standards obsolete and unusable.

Secondly, standards are best applied where long, stable production runs are used. This is not a characteristic of the aerospace industry. Thirdly, this approach requires more personnel, data, and time than the other approaches. Using this approach requires as many as 4500 estimates per airframe (Batchelder, et al, 1969:5). Lastly, for many purposes, industrial engineering estimates have been found to be less accurate than those made statistically (Batchelder, et al, 1969:5). This approach is best used in the latter stages of development when engineering studies have been made and, perhaps, some components or prototypes built.

The analogy approach attempts to compare a previous system with a planned system and by contrasting similarities and differences a cost estimate for the new system is obtained. The analogy approach is used in the aerospace industry quite often since a firm's knowledge of production requirements and costs is usually limited to its own history. This approach requires a great deal of judgment and relies on past experience and expertise to arrive at a satisfactory cost estimate. It is most useful when the future product is essentially the same as that of the past or present period.

The parametric cost estimating approach uses statistical theory to relate historical data on past system costs to future system costs. Parametric models are designed to be used at a time when very little is known about the aircraft (Large, et al, 1977:1). This approach is widely used, as are the others, by government and industry and will constitute the basis for this study. It is best applied in the conceptual stages of a new system when the least is known about that system.

In applying the parametric approach, cost estimating relationships (CER's) that will accurately predict future weapons systems costs are developed. The procedure is to decide what variables are logically or theoretically related to cost and then to look for patterns in the data which suggest a relationship between cost and the variables (Batchelder, et al, 1969:34). The variables can be related to cost in a "causation" cost model or a "correlation" cost model. This distinction can be important. Both models have the property that cost = f (characteristics) but only a causation model can be transformed where characteristics = f^{-1} (cost). Confusing the two can lead to erroneous and embarrassing results (Batchelder, et al, 1969:57). The parametric cost approach must not be used blindly but must be tempered with experience and understanding. Estimating relationships can only be derived from historical data, and the past is not always a reliable guide to the future.

HISTORY OF CER DEVELOPMENT

Since World War II, major emphasis has been placed on airframe cost analysis and prediction. Most early studies concluded that three factors, weight, speed, and quantity, best explained the variations in cost among airframes. Weight is given as AMPR weight and is defined in

the <u>Aeronautical Manufacturers' Planning Report</u> as

> "the empty weight of the airplanes less (1) wheels, brakes, tires, and tubes, (2) engines, (3) starter, (4) cooling fluid, (5) rubber or nylon fuel cells, (6) instruments, (7) batteries and electrical power supply and conversion eauipment, (8) electronic equipment, (9) turret mechanism and power operated gun mounts, (10) remote fire mechanism and sighting and scanning equipment, (11) air-conditioning units and fluid, (12) auxiliary power plant unit, (13) trapped fuel and oil."

Speed is maximum speed in knots at best altitude.

A 1971 Rand Corporation study (Levenson, et al, 1971) provided separate estimating equations for engineering, development support, flight test material, and quality control. This study concluded that:

> "The most useful regression equations are exponential in form,....It was found from empirical investigation and past experience that only three basic variables -- aircraft quantity, speed, and AMPR weight -- provide the most useful relationships for most of the cost elements. A fourth variable, production rate, was used with the tooling equation. Little or no predictive improvement is gained by including additional physical and performance variables, by fitting more complicated functions, or by incorporating other characteristics (quantitative or qualitative) of these procurement programs." (Levenson, et al, 1971:2-3).

This seems like too simplistic an explanation to such a complicated problem as airframe cost prediction. Therefore, Rand Corporation undertook additional studies. With a broadened data base and more sophisticated prediction techniques, a follow-on study was published in 1976 (Large, et al, 1976). In this study all the characteristics below were considered: (Large, et al, 1976:12)

- weight
- speed
- ceiling
- climb rate
- range factor
- thrust-to-weight ratio
- wing loading
- aspect ratio

- static thrust
- lift-to-drag ratio
- load factor
- wetted area
- ratio of gross takeoff weight to airframe unit weight
- wing area
- empty weight minus structure weight
- ratio of wetted area to stress design weight
- ratio of wetted area to wing area

This report states,

> "A determined effort was made...to find additional characteristics that would make an estimating model more flexible and hence better able to deal with characteristics peculiar to individual aircraft. That effort was not productive. The variations in cost that are not explained by weight and speed are not explained by any other objective parameters tested." (Large, et al, 1976:V)

In a separate study, production rate and production cost were considered (Large, et al, 1974). Production rate seems a logical variable since high production rates allow greater use of facilities and greater labor specialization. Materials can be purchased in larger quantities and inventoried for shorter periods. Additionally, increased production reduces overhead rates. This study concluded, however, that the effect of production rate on manufacturing labor, manufacturing materials, tooling, and engineering cannot be predicted with confidence. These findings may have been influenced by the fact that past decisions on production rate were based on military, financial, and political considerations and not on efficiency of production.

LEARNING

The effect of the learning curve on airframe production costs was recognized and applied prior to World War II. Variously called learning, progress, improvement or experience curves, the principle of learning, as set forth by T.P. Wright in 1936, states that as the total quantity of

units produced doubles, the cost per unit declines by some constant percentage. The term "cost per unit" may apply to the average cost of a given number of units in which case a "cumulative average learning curve" is referenced or the term "cost per unit" may apply to the cost of a specific unit in which case a "unit learning curve" is referenced. As an example, if the average cost of producing 200 units is 80 percent of the average cost of producing the first 100 units, an 80 percent cumulative average learning curve ie established. If, on the other hand, the cost of producing the 200th unit is 80 percent of the cost of producing the 100th unit, then a unit learning curve is presented.

Most learning is associated with labor learning and the increased productivity of labor as a result of repetition. While this factor may be most important, other factors contribute to this phenomenon. These include (Batchelder, et al, 1969:94):

1. General improvement in tool coordination, shop organization, and engineering liaison.
2. Development of more efficiently produced subassemblies.
3. Development of more efficient parts-supply system.
4. Development of more efficient tools.
5. Substitution of cast or forged components for machined components.
6. Improvement in overall management.

Simply stated, the learning hypothesis states that as quantity goes up, cost goes down. Mathematically, this can be represented by a power (log-linear) equation of the form

$$c = ax^b$$

where

c = cost (either unit or cumulative average cost depending on treatment)

a = cost of first unit produced

x = number of units

b = exponent measuring the slope of the learning curve.

PRODUCTION FUNCTION MODEL

Another way to view cost estimation is through the production function as set forth in microeconomics. Many estimated cost functions begin with a Cobb-Douglas production function that explains costs in accordance with the theory of the single firm. A generalized Cobb-Douglas production function which considers only two categories of input - capital and labor - appears in the form

$$q = aK^{e_1}L^{e_2}$$

This function is homogeneous of degree $e_1 + e_2$. This sum also defines the returns to scale parameter. Returns to scale describe the change in output to a proportionate change in input. Returns to scale may be constant, increasing or decreasing. If output and input change in the same proportion, returns to scale are constant and the function is homogeneous of degree one. If output increases at a faster rate than input, increasing returns to scale are evidenced and the function is homogeneous of degree greater than one. Similarly, if output increases at a slower rate than input, the production function exhibits decreasing returns to scale and homogeneity is less than one. A production function over its entire range may experience all these cases of returns to scale.

This generalized Cobb-Douglas production function can be augmented with the learning hypothesis and time value of money concept to derive a cost estimating model.

WOMER MODEL

While past empirical studies conclude that production rate contributes little explanatory value to a CER, economic models of the production to order variety frequently conclude that production rate does influence unit cost. Womer has developed a model which allows production rate to be a decision variable in the contractors' production planning. This model provides for changes in production rate as the program progresses, thereby optimizing the production plan. This model forms the basis for this thesis effort.

The development of the model begins with a production function of the form

$$q(t) = A\, Q^{\delta}_{(t)}\, X^{1/\alpha}_{(t)} \tag{1}$$

where

$q(t)$ = output rate of the program at t

A = a constant

$Q(t) = \int_0^t q(\tau)d\tau$ = cumulative production experience at t

$x(t)$ = rate of resource use at t

δ = a parameter describing learning

α = a return to scale parameter

Solving (1) for x(t) and dividing by q(t) gives

$$x(t)/_{q(t)} = A^{-\alpha} q(t)^{\alpha-1}\, Q^{-\alpha\delta}_{(t)} \tag{2}$$

Next, considering input prices and the discount rate to be exogenous variables, a program cost is determined by

$$C = \int_0^T X(t)e^{-pt}dt \tag{3}$$

where

C = discounted program cost

T = total length of the production period

$x(t)$ = rate of resource use at t

p = discount rate

(The present value concept is incorporated by use of the continuous discount factor e^{-pt}, where p is the discount rate (Dunne, 1975:11). Combining (1) and (3) leads to a problem in optimal control (Intrilegator, 1971:292-304) characterized as

$$\min \int_0^T X(t)e^{-pt}dt$$

$$St \quad q(t) = A\, Q^{\delta}_{(t)}\; x^{1/\alpha}_{(t)}$$

$$x(t) \geqq 0 \quad Q(0) = 0 \quad Q(T) = V$$

where

T = total length of the production period

$x(t)$ = rate of resource use at t

p = discount rate

$q(t)$ = output rate of the program at t

A = a constant

$Q(t)$ = cumulative production experience at t

δ = a parameter describing learning

α = a return to scale parameter

V = volume of output produced by T

After an appropriate transformation, several substitutions and mathematical operations (Womer,....:4-6), the total discounted cost of the program in terms of V and T is

$$C(V,T) = [p/(\alpha-1)]^{\alpha-1}(1-\delta)^{-\alpha}\ AV^{\alpha(1-\delta)}[e^{pT/(\alpha-1)}-1]^{1-\alpha} \tag{4}$$

Integrating (4) from 0 to t where t<T yields

$$C(t/V,T) = [p/(\alpha-1)]^{\alpha-1}(1-\delta)^{-\alpha}AV^{\alpha(1-\delta)}[e^{pT/(\alpha-1)}-1]^{-\alpha}[e^{pt/(\alpha-1)}-1] \tag{5}$$

which is the time path of cumulative discounted cost given V and T. Womer refers to (4) as the planning situation and (5) as the production situation. Equation (4) is used in planning to determine optimal levels for volume and time. In Equation (5) volume and time are given. Thus the cost at any time, t, can be determined.

After an appropriate, simplifying substitution (Womer...:7), (5) is changed to read

$$C(t/V,T) = [p/(\alpha-1)]^{\alpha-1}(1-\delta)^{-\alpha}AV^{(\alpha-1)(1-\delta)}[Q(t)]^{1-\delta}[e^{pT/(\alpha-1)}-1]^{1-\alpha} \tag{6}$$

This model joins the concept of learning to the production function. Implicit in the use of this model are the following assumptions:

- The production function is homogeneous.
- The contractor experiences decreasing returns to scale.
- The contractor experiences learning by doing.
- Prices of resources are fixed.
- The contractor has flexibility in resource use.
- The program is defined by volume of output (V) available at time (T).
- The contractor has no prior production experience.
- The goal is to minimize the discounted cost of producing volume V at time T.

The model presented here and several variations as derived in Chapters II and IV will be augmented with the traditional CER variables of weight and speed and it is these augmented forms of the model that are used for the analysis in this thesis.

The objective of this thesis is to determine if the Womer model is significantly better in explaining and predicting aircraft airframe costs than the more simple models of Large and Levenson which contain only weight, speed, and quantity as decision variables. In meeting this objective, data was gathered on a selected group of aircraft. Using the data, nonlinear least squares regression analysis was used to determine values for the unknown parameters in the model. These parameters were tested for statistical significance and the predictive ability of the model was determined. The findings can be found in the conclusion of this thesis.

Dunne, in 1975, attempted the first validation of the Womer model. His study, however, fell short of its objective in two areas. First, appropriate values for volume and time, as specified in the model, could not be obtained. He used actual volume and time in place of planned volume and time. Secondly, he was unable to use nonlinear least squares regression in his analysis. He, therefore, transformed the Womer model into a form that could be analyzed linearly, and, in doing so, some of the original meaning of the model was lost. This study overcomes, to a large extent, the two problems encountered by Dunne.

Chapter II contains the methodology of this thesis including treatment of the data base and method of solution. Chapter III is a review of the statistical methods used in the analysis which is contained in Chapter IV. Chapter V deals with the problem of model validity and Chapter VI contains concluding remarks.

II. METHODOLOGY

DATA BASE

Data is the essence of any analysis. The data can greatly affect the outcome and, therefore, should be picked with care. Far too often, however, the data required has not been collected or is not available. In such cases, less than ideal data must be used. The use of such data, or any data for that matter, for production studies or economic predictions raises questions of comparability and consistency (Dunne, 1975:15).

To further complicate the collection of data, accounting practices differ between companies, and even in the same company over a period of time. Additionally, terminology and methods of data collection are not standardized.

With these problems and complications in mind, an attempt must be made to reconcile the discrepancies and gather the best available data, bearing in mind the limitations of that data.

First, the items of data needed must be determined. These items are dictated by the model and include program cost (C(t)), cumulative production experience at time t (Q), time horizon for the production program (T), and volume of output to be produced by T (V). Additionally, the AMPR weight (W) and speed (S) of each aircraft will be needed.

Having determined the items of data needed, the aircraft to include in the data base must be chosen. The data base was originally planned to consist of the sixteen aircraft listed in Table I. These aircraft constitute a somewhat homogeneous set in that they are jet aircraft employing post World War II technology. They range in speed from 578 knots to 1,262 knots and weigh from 5,072 pounds to 32,458 pounds.

TABLE I

Aircraft in Original Data Base

A-3	A-7	F-101	F-106
A-4	F-4	F-102	F-111
A-5	F-14	F-104	T-38
A-6	F-100	F-105	T-39

With the specific data requirements in mind, a search for the necessary data was conducted. No single source contains all the data desired. A possible source considered was the airframe contractors. A letter was sent to all the contractors involved requesting their planned production schedule at the time the first production contract was signed. Those contractors contacted and their respective airframes were Fairchild Republic Company (A-10, F-105), General Dynamics Convair Division (F-102, F-106), General Dynamics Fort Worth Division (F-111), Grumman Aerospace Corporation (A-6, F-14), Lockheed Aircraft Corporation (F-104), McDonnell Douglas Corporation (A-3, A-4, F-4, F-15), Northrup Corporation (T-38), Rockwell International Corporation (A-5, F-100, T-39), and Vought Corporation (A-7). A sample letter is included as Appendix A to this thesis.

Response to these letters was less than encouraging. In many cases, either no data was received or it was not available. In other cases where data was received, it was either incomplete or unusable. Therefore, alternate sources for data were sought.

A second source of data was the cost records in the Cost Library (ACCM) of the Aeronautical Systems Division (ASD) at Wright-Patterson Air Force Base. Specifically, the library has available the data used by Rand in report R-1693 (Large, et al, 1976). Rand Corporation, in compiling this data, studied the available records, reviewed accounting procedures, and reworked the data to provide a consistent base. The

data need only be indexed to a given year to provide comparability. This was done by J. Watson Noah Associates, Inc. (Noah, et al, 1977) in their Aircraft Cost Handbook. From this publication were drawn values for C (cost) and Q (quantity). The index year used was 1975. Also, W (weight) and S (speed) were taken from this handbook.

All that remained was to find values for V (volume) and T (time). One possible source was the aircraft contracts. Unfortunately, many of the contracts have been destroyed and records of planned volume and time were not kept. The ASD historian could not provide the needed data. Neither could the research department of the Air Force Museum at Wright-Patterson Air Force Base. As a compromise for the needed data, planned acceptance rates and times as listed in Assistant Secretary of Defense (Program Analysis and Evaluation) report Acceptance Rates and Tooling Capacity for Selected Military Aircraft (Office of Assistant Secretary of Defense (Program Analysis and Evaluation), 1974) were used. This report furnished values for V (volume) and T (time) in this study.

Unfortunately, all the required data were not available for all sixteen aircraft as listed in Table I. Those aircraft for which all data were available and those used in this study are listed in Table II.

TABLE II

Aircraft in Actual Data Base

A-4	A-7	F-105
A-5	F-4	F-111
A-6	F-104	T-38

A listing of the complete data gathered for this study can be found in Appendix B.

MODEL REFINEMENT

The model, as expressed in (6) and as restated below, must be refined and surrogate variables introduced as needed to facilitate solution.

$$C(t/V,T) = [p/(\alpha-1)]^{\alpha-1}(1-\delta)^{-\alpha}AV^{(\alpha-1)(1-\delta)}[Q(t)]^{1-\delta}[e^{pT/(\alpha-1)}-1]^{1-\alpha}$$

The first two terms of the model $[(p/\alpha-1)^{\alpha-1}(1-\delta)^{-\alpha}]$ will constitute the first unknown parameter and will thus be known as β_1. The constant term, A, can be thought of as a function of weight and speed (A=f(W,S)) and will hence be replaced by $W^{\beta_2}S^{\beta_3}$. The exponent of Q, $(1-\delta)$, will be β_4. The exponent of V,$(\alpha-1)(1-\delta)$, will be β_5. The exponent of the last term, $(1-\alpha)$, will be β_6. Examination of these newly assigned exponents will reveal that $\beta_5 = \beta_4\beta_6$. Rewriting the model in (6) with the above substitution yields

$$C(t/V,T) = \beta_1 W^{\beta_2}S^{\beta_3}Q^{\beta_4}V^{\beta_5}(e^{pT/-\beta_6}-1)^{\beta_6} \tag{7}$$

where

C = discounted program cost at time t.

W = AMPR weight

S = maximum speed at best altitude

Q = cumulative production experience at time t

V = volume to be produced at T

T = time horizon of the production program

p = discount rate

This form of the model calls for discounted program cost (C) to be used. The discounted cost is not available, and, due to data restrictions cannot be completed. Therefore, undiscounted costs, as contained in the Rand data, will be used, but the symbol, C, will be retained.

An undiscounted cost model can be derived in much the same way as the discounted cost model was derived in Chapter I.

Let C* be the undiscounted cost. Then

$$C^* = \int_0^T X(t)dt \tag{8}$$

As Womer has shown (Womer,....:4),

$$X(t) = A^{-\alpha}Z^{\alpha}(t)$$

Therefore, substituting into (8)

$$C^* = \int_0^T A^{-\alpha}Z^{\alpha}(t)dt$$

But $A^{-\alpha}$ is merely a constant (let it be a) and Womer (Womer....:4) has shown that

$$Z(t) = Q^{-\delta}(t)q(t)$$

Then

$$C^* = a\int_0^T q^{\alpha}_{(t)} \quad Q^{-\alpha\delta}_{(t)} \; dt$$

Following the substitution by Womer (Womer....:5)

$$C^* = a\int_0^T V^{\alpha(1-\delta)}(e^{pT/(\alpha-1)}-1)^{-\alpha} \; e^{\alpha pt/(\alpha-1)}dt$$

$$= aV^{\alpha(1-\delta)}(e^{pT/(\alpha-1)}-1)^{-\alpha}(e^{\alpha pt/(\alpha-1)}-1) \tag{9}$$

This model must also be refined and surrogate variables introduced. Let $a = \beta_1 W^{\beta_2} S^{\beta_3}$, $\alpha(1-\delta) = \beta_4$, $(\alpha-1) = \beta_5$ and $\alpha = \beta_6$ (notice that $\beta_5 = \beta_6-1$). Then (9) can be rewritten

$$C^* = \beta_1 W^{\beta_2} S^{\beta_3} V^{\beta_4}(e^{pT/\beta_5}-1)^{-\beta_6}(e^{\beta_6 pt/\beta_5}-1) \tag{10}$$

This is one form of the undiscounted cost model used in this thesis.

An alternate form of the undiscounted cost model involves a simple substitution. From Womer's expression for Q (Womer, ...:5), it can be shown that

$$e^{pt/\alpha-1} = [(Q/V)^{1-\delta}(e^{pT/\alpha-1}-1)] + 1$$

Substituting this expression into (9) yields

$$C^* = aV^{\alpha(1-\delta)}(e^{pT/\alpha-1}-1)^{-\alpha}[\{(Q/V)^{1-\delta}(e^{pT/\alpha-1}-1)+1\}^{\alpha}-1] \qquad (11)$$

This form of the undiscounted cost model eliminates t (little t) as a decision variable.

This model must again be refined and surrogate variables introduced. Let $a = \beta_1 W^{\beta_2} S^{\beta_3}$, $\alpha(1-\delta) = \beta_4$, $(\alpha-1) = \beta_5$, $\alpha = \beta_6$, and $(1-\delta) = \beta_7$. (Notice that $\beta_4 = \beta_6\beta_7$ and $\beta_5 = \beta_6-1$). Substituting these new parameters into (11) yields

$$C^* = \beta_1 W^{\beta_2} S^{\beta_3} V^{\beta_4}(e^{pT/\beta_5}-1)^{-\beta_6}[\{(Q/V)^{\beta_7} e^{pT/\beta_5}-1)+1\}^{\beta_6}-1] \qquad (12)$$

The cost models, as contained in (7), (10), and (12), are used in performing the analysis reported in Chapter IV.

METHODS OF SOLUTION

The models, as stated in this thesis, are to be evaluated using least squares regression analysis. Regression analysis can be divided into two subdivisions, linear and nonlinear. A linear model (linear in the parameters) is of the form

$$Y = \beta_1+\beta_2X_1+\beta_3X_2+\ldots\ldots+\beta_{k+1}X_k + \varepsilon \qquad (13)$$

where

Y = dependent variable

$X_i, i=1,k$ = independent variables

$B_j, j=1,k+1$ = unknown parameters to be estimated

ε = error term

Any model that is not of the form given in (13) is nonlinear. The models given in (7), (10), and (12) are nonlinear models; that is, nonlinear in the parameters.

There are essentially three ways to solve a nonlinear model. These are linearization, steepest descent and Marquardts' compromise.

The linearization method makes use of a Taylor series expansion of the model, dropping the nonlinear terms. This linear representation of the model is then regressed using linear regression techniques yielding an estimate of the unknown parameters. This procedure is iterated until the sum of squares reaches a minimum. For example, consider the model

$$Y = f(X,\beta) + \varepsilon \tag{14}$$

where

Y = dependent variable

X = set of independent variables

β = set of parameters

ε = error term

Let β_o be an initial estimate for β. This estimate may be an intelligent guess or preliminary value based on theory or available data. A Taylor series expansion of the function $f(X,\beta)$ using the initial values for β can be written

$$f(X,\beta) = f(X,\beta_o) + \sum_{i=1}^{k} \left[\frac{\partial f}{\partial \beta_i} \right]_{\beta=\beta_o} (\beta_i - \beta_{io}) \qquad (15)$$

Since, in regression, the set of independent variables, X, are a known quantity, the expression in (14) is now linear in the parameters as the higher order terms of the Taylor series expansion are ignored. Let

$$F^o = f(X,\beta_o), \text{ a constant term}$$

$$Z_i^o = \left[\frac{\partial f}{\partial \beta_i} \right]_{\beta=\beta_o}, \text{ a constant term}$$

$$\beta_i^o = \beta_i - \beta_{io}$$

then (15) can be expressed

$$f(X,\beta) = F^o + \sum_{i=1}^{k} Z_i^o \beta_i^o \qquad (16)$$

Substituting (16) into (14) and rearranging yields

$$Y - F^o = \sum_{i=1}^{k} Z_i^o \beta_i^o + \varepsilon \qquad (17)$$

which has the standard form of the linear regression model. This expression may be evaluated

$$\beta_i^o = (Z'Z)^{-1} Z'(Y - F^o)$$

giving a revised estimate of the parameters, β. This new estimate is substituted in the sum-of-squares function

$$SS(\beta) = \sum_{i=1}^{k} [Y_i - f_i(X,\beta)]^2 \qquad (18)$$

and the above process is iterated until the sum-of-squares function is minimized or until the relative change in the parameters between the current iteration and the previous iteration is less than some small, predetermined tolerance.

There are several drawbacks to this method (Draper, et al, 1966: 269). First, it may converge very slowly and a great number of iterations may be necessary to reach the stopping criteria. Second, it may oscillate, continually reversing direction and often increasing, as well as decreasing, the sum-of-squares function. Third, it may not converge at all.

The steepest descent method begins with the sum-of-squares function given in (18). The method starts with the first partial derivatives of the sum-of-squares function, $SS(\beta)$, and evaluates them at the starting point, β_o. This set of numbers is the gradient and always points away from a minimum in its vicinity (local minimum). A move is made from the initial parameter estimates, β_o, in the direction of the negative gradient, thus moving toward a minimum. If the estimate of the parameters is corrected by the negative gradient at each iteration, a minimum will be reached eventually. At the minimum, the gradient will be zero. This procedure will converge, but it may do so at a very slow pace (Draper, et al, 1966:271).

"Marquardts' method represents a compromise between the linearization (or Taylor series) method and the steepest descent method and appears to combine the best features of both while avoiding their most serious limitations" (Draper, et al, 1966:272). Basically the method starts at a certain point in the parameter space, β. The steepest descent method is applied and a vector direction, δ_g, is obtained, where g stands for gradient. δ_g represents the best local direction to move, but may not be the best overall direction. The best direction, however, lies within 90^o of δ_g. The linearization method yields another vector, δ. The

angle ϕ between δ_g and δ lies in the range $80^0 < \phi < 90^0$. The Marquardt algorithm interpolates between δ_g and δ finding the optimum path to minimize the sum-of-squares function (Draper, et al, 1966:272-273).

SPSS

The Statistical Package for the Social Sciences (SPSS) is a set of computer subprograms to facilitate the statistical analysis of data. One of these subprograms is called NONLINEAR and is used for nonlinear regression analysis. This subprogram has the option of applying two methods, linearization, known as the Gauss' method, and Marquardts' method, to problems of nonlinear regression.

This subprogram was used extensively in this study and represents the principle means of achieving nonlinear regression.

III. STATISTICAL BACKGROUND

This chapter has two main purposes. First, it discusses the treatment of the error term in the models used. Secondly, it outlines the statistical procedures used in evaluating the regression analysis.

Treatment of Error Term

The models used in this thesis are of the nonlinear type with additive error term. Such models are of the general form

$$Y = f(X, \beta) + \varepsilon$$

where

Y = dependent variable

$X_i, i = 1,k$ = independent variables

$\beta_i, j = 1,k+1$ = parameters to be estimated

ε = error term

The error term is considered to be normally distributed with mean equal to 0 and variance equal to σ^2. This is usually written: $\varepsilon \quad N(0,\sigma^2)$. "Calculation of the statistical properties of the regression equations requires an additive error" (Levenson, et al, 1971:37). Hence, this form of the error term has been adopted.

Statistical Procedures

There are two problems associated with the statistical evaluation of the models presented in this thesis. First, "the usual tests which are appropriate in the linear model case are, in general, not appropriate when the model is nonlinear" (Draper, et al, 1966:282). There are no

specific statistical tests of the nonlinear model found in this research. Therefore, the usual linear tests will be used, bearing in mind that the test results are not absolute but merely guides as to the statistical merit of the particular solutions being tested (Robinson, 1977:21).

The second problem is that

> "the standard statistics that are used to evaluate regression models (e.g., coefficient of correlation, standard error estimate) do not give direct statements regarding the ability of the model to predict future observations. They indicate how well the model fits the data" (Levenson, et al, 1971:39).

The solution to this problem requires a measure that makes a probability statement about the confidence that can be placed in a prediction. Therefore, use will be made of prediction intervals. "Prediction intervals are limits within which the value of a single future observation lies with a specified probability" (Levenson, et al, 1971:39).

The statistical tests used in this thesis are defined below. These tests give a relative relationship of the "goodness of fit" of the model and establish relative error bounds on predictions. The statistics include:

MSE

R^2

t statistic

F statistic.

<u>Mean Squared Error</u> is an unbiased estimator of the model's variance. It is obtained by dividing the sum-of-squares for error (SSE) by the degrees of freedom.

$$MSE = \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n-k} = \frac{SSE}{n-k}$$

where

$Y_i, i=1,n$ = dependent variable

$\hat{Y}_i, i=1,n$ = regression estimator of Y_i

n = number of observations

k = number of parameters estimated

SSE = sum-of-squares for error

In model evaluation, a small variance is desired and indicative of good explanatory and predictive ability.

The Coefficient of Determination measures how well the explanatory variables account for the variations in the actual cost data. That is, R^2 measures the proportion of total variation about the mean of Y that is explained by the regression. Thus

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} = 1 - \frac{SSE}{SST}$$

where

Y_i, $i=1,n$ = dependent variable

$\hat{Y}_i$, $i=1,n$ = regression estimator of Y_i

$\bar{Y}$ = average of dependent variables

SSE = sum-of-squares for error

SST = total sum of squared deviations about the mean

NOTE: In a model containing an additive constant term, SST=SSR+SSE where SSR = sum-of-squares for regression. In a model without an additive constant term, this relationship does not hold. The models used in this thesis do not have an additive constant term.

Therefore, in this context, R^2 takes on a different meaning and can only be used as a basis for comparison.

t Statistic

The Students' t statistic is the ratio of a standard normal random variable divided by the square root of a X^2 random variable which was divided by its degree of freedom. This statistic is used to determine the significance of an individual parameter and, also, is used in the computing of confidence intervals and prediction intervals. The test for significance of an individual parameter is as follows:

$$H_o: \beta_i = 0$$
$$H_1: \beta_i \neq 0$$

$$T = \frac{b_i - \beta_i}{\sqrt{S^2(X^1X)^{-1}_{ii}}}$$

$$\text{Reject } H_o \text{ if } |T| \geq t_{\alpha/2}$$

where

b_i = the parameter being tested

$S^2(X^1X)^{-1}_{ii}$ = ii entry of the variance-covariance matrix

$t_{\alpha/2}$ = Students' t statistic for two tailed test

α = confidence level of the test.

To construct a $(1-\alpha)$ confidence interval for b_i, the statistic is used

$$b_i \pm t_{\alpha/2}\sqrt{S^2(X^1X)^{-1}_{ii}}$$

Similarly, a $(1-\alpha)$ prediction interval for Y (cost) is constructed

$$\hat{Y} \pm t_{\alpha/2}\sqrt{S^2_*}$$

where

$$S_*^2 = S^2 X_* (X'X)^{-1} X_*' + S^2$$

X_* = values for independent variables for which a prediction is desired.

This form of the prediction interval assumes a one to one correspondence between the number of parameters and the number of independent variables. In a nonlinear model, this correspondence does not necessarily hold. Therefore, an alternative method is necessary. The first partial derivatives with respect to the parameters are taken. Substituted into the first partials are the values for the independent variables for which a prediction is desired (X_* from above). Additionally, the values for β determined by the regression analysis are substituted. This results in a vector of scalars representing the first partials at the point X_*. This vector is $\underline{g}$. Let

$$G = \sqrt{\underline{g}' V \underline{g} + S^2}$$

where

$\underline{g}$ = vector of first partials

V = variance-covariance matrix

S^2 = variance in the model.

The prediction interval is then

$$\hat{Y} \pm t_{\alpha/2}\, G$$

where

$t_{\alpha/2}$ is defined above.

In using the t statistic in the context of a nonlinear model with no additive constant term, it must be assumed that the model is nearly linear in the parameters at the final parameter estimates.

F Statistic

The F distribution is the ratio of two X^2 random variables divided by their respective degrees of freedom. It is used to compare the relative merit of the full model against that of a restricted model in which some of the β_i's are zero. The test is stated

H_o : restricted model is better model

H_1 : restricted model is not better model

$$F_o = \frac{\frac{SSE_r - SSE}{r}}{\frac{SSE}{n-k}} \sim F\,(r,\, n-k)$$

Reject H_o if $F_o > F$

where

SSE_r = sum-of-squares for error of a model in which linear restrictions have been placed on the parameters

SSE = sum-of-squares for error in the same model without the restriction

r = the number of linear restrictions imposed

n = number of data points

k = the number of parameters in unrestricted model.

IV. ANALYSIS

The analysis concerns the comparison of the Womer model as represented in equation (7) to several other models of the same type as they are regressed on the same data base. The Womer model will be referred to as Model I in this chapter.

When the Womer model, as stated in (7), is linearly constrained such that β_5 and β_6 are zero, the model becomes

$$C = \beta_1 W^{\beta_2} S^{\beta_3} Q^{\beta_4} \tag{19}$$

This is the model used by Levenson (Levenson, et al, 1971) and will form the base case against which the Womer model is compared. This is Model II.

Substituting T for the exponential term of the Womer model yields

$$C = \beta_1 W^{\beta_2} S^{\beta_3} Q^{\beta_4} V^{\beta_5} T^{\beta_6} \tag{20}$$

which is a model that can be reduced to log-linear form and analyzed by linear means. This is the model used by Dunne (Dunne, 1975) and is referred to as Model III in this chapter. The interpretation of the parameters β_1 through β_5 is the same as in the Womer model. The interpretation of β_6 is unclear and this model is only considered due to its simplicity and previous use by Dunne.

The undiscounted cost versions of the Womer model constitute Models IV and V. Equation (10) will be called Model IV and equation (12) will be called Model V. All five models are summarized in Table III.

TABLE III

MODEL SUMMARY

MODEL I (Womer)

$$C = \beta_1 W^{\beta_2} S^{\beta_3} Q^{\beta_4} V^{\beta_5} (e^{-pT/\beta_6} - 1)^{\beta_6}$$

where $\beta_5 = \beta_4 (-\beta_6)$

MODEL II (Levenson)

$$C = \beta_1 W^2 S^{\beta_3} Q^{\beta_4}$$

MODEL III (Dunne)

$$C = \beta_1 W^{\beta_2} S^{\beta_3} Q^{\beta_4} V^{\beta_5} T^{\beta_6}$$

MODEL IV (Womer Undiscounted)

$$C^* = \beta_1 W^{\beta_2} S^{\beta_3} V^{\beta_4} (e^{pT/\beta_5} - 1)^{-\beta_6} (e^{\beta_6 pt/\beta_5} - 1)$$

where $\beta_5 = \beta_6 - 1$

MODEL V (Womer Undiscounted-Alternate Form)

$$C^* = \beta_1 W^{\beta_2} S^{\beta_3} V^{\beta_4} (e^{pT/\beta_5} - 1)^{-\beta_6} \left[\{ (Q/V)^{\beta_7} (e^{pT/\beta_5} - 1) + 1 \}^{\beta_6} - 1 \right]$$

where $\beta_4 = \beta_6 \beta_7$

$\beta_5 = \beta_6 - 1$

where

C = discounted program cost

C*= undiscounted program cost

W = AMPR weight

S = speed at best altitude

Q = cumulative production at time t

T = time horizon for the program

V = volume of output to be produced by T

p = discount rate

The first data base considered for analysis consisted of 52 points. These points were chosen on the basis of the planned acceptance schedule in effect at the time the research and development contract for each aircraft was signed. The time T was measured from the date of the earliest plan listed in the Acceptance Rates and Tooling Capacity for Selected Military Aircraft study (Office of Assistant Secretary of Defense (Program Analysis and Evaluation), 1974). Likewise, the time to produce quantity Q aircraft was measured from the time of the earliest schedule to time of Q actual acceptances. This measure presents some problems. Acceptance lags behind production and, therefore, the time given is probably larger than the actual time taken to produce quantity Q aircraft. No other reliable measures of time to produce quantity Q aircraft were available and this unknown error must be accepted. This set of data is referred to as Data Set I. A listing of the data used is given in Appendix C. The discount rate of 10% was chosen in accordance with Air Force Regulation 178-1 (Department of The Air Force, 1973:11). Table IV summarizes the results of the regression analysis using this data base.

From the results, it is noted that R^2 is quite high which is indicative of a good fit and none of the models distinguishes itself of the basis of R^2. The returns to scale parameter, α, is greater than one but less than two, which is the value to be expected in the aerospace industry. This value represents moderately decreasing returns to scale. The slope of the learning curve for all the models except IV is relatively close to the empirically derived 80% slope which characterizes the aerospace industry (Large, et al, 1976:29). It should also be noted that the small magnitude of β_1 is due to the scaling of the independent variable.

TABLE IV

RESULTS OF REGRESSION ANALYSIS
DATA SET I

Model Parameters	I	II	III	IV	V
β_1	.00002	.00002	.00021	.00687	.00002
β_2	1.2186	1.0902	1.2029	1.6236	1.1435
β_3	.4438	.4970	.4360	-.9573	.4448
β_4	.6448	.6529	.6445	.3785	.6916
β_5	.0528		.0471	.7055	.3283
β_6	-.0820		-.6485	1.7055	1.3283
β_7					.5206
α	1.0820	-	1.0731	1.7055	1.3283
δ	.3552	.3471	.3555	.7781	.4794
Learning Curve Slope	78.18%	78.61%	78.16%	58.31%	71.72%
Statistical Results					
df	47	48	46	47	47
SSE	5.2E+05	5.8E+05	5.2E+05	1.2E+06	5.3E+05
MSE	1.1E+04	1.2E+04	1.1E+04	2.6E+04	1.1E+04
R^2	.99	.99	.99	.97	.99

Due to its small size, β_1 was not statistically different from zero when 95% confidence intervals were computed for all the parameters. In Model I, β_6 was also not statistically different from zero at the α = .05 confidence level.

The F test, as presented in Chapter III, was applied to Models I and II with Model II being the restricted model. With an F_O = 4.55, the null hypothesis that the restricted model is the better model is rejected at a confidence level of α = .05. In comparing Model II to Model III, F_O = 2.64 and the null hypothesis that the restricted model is the better model is not rejected.

Additional regressions were made with the discount rate as a decision parameter. With Model I, the optimal value of the discount rate to minimize the sum of squares was 8.57%. Applying the same procedure to Model IV produced a discount rate of 15.36%. Model V produced inconclusive results.

To be most effective, the Womer model requires that actual production follow the designated plan. The existing data set of 52 points was reexamined and those points where t (time to product quantity Q airframes) exceeded T (time horizon for the program) were immediately eliminated. The remaining points were then examined for "reasonableness". "Reasonableness" was determined by looking at the average planned production per month (V/T) and average actual production (Q/t). The rate of production to complete V (volume of output to be produced by T) was then calculated ((V-Q)/T-t)). If the rate of production to complete V was near the planned production per month or if the rate of production to complete V did not exceed 33, the point was considered reasonable.

This procedure reduced the number of data points to 30. This set is Data Set II and a listing is provided in Appendix D.

This new data set was regressed and analyzed in the same manner as Data Set I. The discount rate remained at 10%. The results of this analysis appear in Table V.

Upon examining the Table, the change in discount rate for Model I is immediately apparent. This model on Data Set II would not converge for discount rates less than 15%. No plausible reason can be given for this failure to converge. The SPSS algorithm for nonlinear least squares does seem to be sensitive to the initial starting values and contents of the data base.

The overall performance of the models on Data Set II was not too different from that on Data Set I except for MSE. MSE was one magnitude lower on Data Set II which means less unbiased variance. The results of Data Set II should provide better explanatory and predictive ability for programs that follow this plan.

The F test was again applied to Models I and II yielding F_0 of 3.48 and Models II and III yielding F_0 of 2.74. In neither case can the null hypothesis that the restricted model is the better model be rejected at a confidence level of $\alpha = .05$. This leads to conflicting results. The improvement of the Womer model over the restricted model on Data Set II is not statistic significant at the 5% level yet it is on Data Set I. Model III likewise in either case. Unfortunately, due to the lack of a one to one correspondence between the parameters in Models IV and V and the restricted model, as represented by II, Models IV and V cannot be subjected to the F test.

Left unanswered is the question as to which model is best. Perhaps an analysis of the predictive ability of the models, as presented in Chapter V, will shed some light on this issue.

TABLE V

RESULTS OF REGRESSION ANALYSIS
DATA SET II

Model Parameters	I*	II	III	IV	V
β_1	.00007	.00019	.03220	.00246	.0009
β_2	1.1728	.8264	1.7635	2.1118	.9119
β_3	.4179	.6185	.1422	-1.3818	.5859
β_4	.6215	.5985	.6739	.3630	.6228
β_5	.0019		.0557	.2611	.2148
β_6	-.0031		-2.5779	1.2661	1.2148
β_7					.5126
α	1.0031	-	1.0826	1.2611	1.2148
δ	.3785	.4015	.3261	.7122	.4874
Learning Curve Slope	76.92%	75.70%	79.77%	61.03%	71.33%
Statistical Results					
df	25	26	24	25	25
SSE	1.3E+05	1.4E+05	1.2E+05	2.0E+05	1.4E+05
MSE	5.1E+03	5.5E+03	4.9E+03	7.8E+03	5.7E+03
R^2	.96	.95	.97	.94	.96

*Discount Rate of 15% used.

V. MODEL VALIDITY

The true test of a cost estimation model is in its ability to predict future costs. A model is valid only to the extent that it can accurately predict. Chapter IV gave an indication as to how well the various models fit the data. In this chapter the models will be used in an attempt to estimate the costs of other airframes.

Models and Airframes Used

Based on an assessment of the models presented in Chapter IV, three were chosen for prediction analysis. These models were chosen as a result of their statistical properties and are representative of all the models used in this study. The models selected were Model I, Womer discounted cost model, Model II, Levenson or constrained Womer model, and Model V, Womer undiscounted cost model.

The airframes chosen for prediction analysis were the F-14 and F-15. These airframes were chosen for two reasons. First, they are the newest, state of the art aircraft in the Navy and Air Force inventory. They are representative of future aircraft for which cost predictions may be sought. Secondly, and perhaps more importantly, sufficient data was available on these aircraft to justify a prediction analysis. The data came from two sources. Cost, quantity, weight, and speed information was furnished by J. Watson Noah Associates, Inc., through their Aircraft Cost Handbook (Noah, et al, 1977). Values for volume and time were taken from production plans and delivery schedules furnished by the contractors. These schedules can be found in Appendix E. The data used appear in Table VI.

TABLE VI

PREDICTION DATA

AIRCRAFT	ACTUAL COST (Mil. 75$)	WEIGHT (lbs)	SPEED (knots)	QUANTITY	VOLUME	TIME (months)
F-14	1050.833	26,016	1370	86	469	111
F-15	1032.353	17,364	1319	112	749	116

Prediction Results

Cost predictions were made for the F-14 and F-15 airframes using the three models listed above. Predictions are made using the parameters determined by regression analysis on both Data Sets I and II. The prediction results are summarized in Table VII. Prediction error is the absolute difference between the actual and predicted cost.

In Table VII, the prediction values for Model I, Data Set II are missing. This is due to the fact that Model I would not converge on Data Set II. No parameters were estimated and, therefore, no prediction could be made.

Examination of the computed prediction errors reveals a wide disparity in the predictive ability of the models, given the data set and aircraft. The relatively accurate predictions of the F-14 are nullified by the inaccurate predictions of the F-15. Both results are considered alarming. No plausable reason is seen for this disparity in results.

Due to the disparity in results, average prediction errors were computed for each model and data set across the two airframes. These results are displayed in Table VIII.

TABLE VII

PREDICTION RESULTS

MODEL	DATA SET	ACFT	PREDICTED COST	ACTUAL COST	PREDICTION ERROR
I	I	F-14	1091.650	1050.833	40.817
		F-15	764.564	1032.353	267.789
II	I	F-14	1209.588	1050.833	158.755
		F-15	907.654	1032.353	124.699
	II	F-14	1060.150	1050.833	9.317
		F-15	868.418	1032.353	163.935
V	I	F-14	1158.904	1050.833	108.071
		F-15	859.332	1032.353	173.021
	II	F-14	1068.298	1050.833	17.465
		F-15	850.332	1032.353	185.021

All costs expressed in millions 75%.

TABLE VIII

AVERAGE MODEL - DATA SET

PREDICTION ERRORS

		DATA SET I	DATA SET II
MODEL	I	154.303	-
	II	141.727	86.626
	V	140.546	101.243

This Table shows Model V to be the best on Data Set I, but not by much. Model II is clearly better on Data Set II. The striking difference is in the predictive ability of the two data sets. Data Set II is clearly superior to Data Set I. This could certainly be attributed to the care with which Data Set II was chosen.

Having made this comparison of averages, an additional step was taken. The average for each model, each data set, and each airframe was taken. These averages were determined by taking the average of the prediction errors across all the observations for that particular item of interest. The results are contained in Table IX.

TABLE IX

AVERAGE PREDICTION ERROR

MODEL		DATA SET		AIRFRAME	
I	154.303	I	145.525	F-14	66.885
II	114.177	II	93.185	F-15	182.293
V	120.145				

Table IX shows that Model II is the best, but is not that much better than Model V. Model II is clearly better than Model I. This can be attributed to the fact that Model I calls for discounted cost and undiscounted cost was used. Data Set II is again superior to Data Set I and this result is again not surprising. The Womer models assume that the production plan used is actually being followed. This is more likely with Data Set II than with Data Set I due to the way Data Set II was assembled. The surprising result is the wide disparity between the two airframes and, again, no plausable reason can be given for this result.

Prediction Intervals

The next logical step in the analysis is the construction of prediction intervals. Before proceeding, however, a few comments regarding prediction intervals are in order.

The percentage $100(1-\alpha)$ is the confidence level of the prediction interval where α is the level of significance. That is to say that if repeated observations on the cost of an airframe, using the same independent variables, were taken, $100(1-\alpha)$ percent of the time these observations would lie within the range set by the $100(1-\alpha)$ prediction interval. This does not mean that there is a $100(1-\alpha)$ probability that the actual value for any particular observation will lie within the interval (Batchelder, et al, (1969:52).

> "Further, prediction intervals are valid outside the range encompassed by the sample data that are used to generate the estimating relationship and the interval only if the estimating relationship is itself valid outside that range." (Batchelder, et al, 1969:52).

With these cautions in mind, 50% prediction intervals were constructed with the results displayed in Table X.

The 50% prediction interval was chosen due to the extreme width of the intervals. The intervals are wide because "the formula for the prediction interval is such that the width of the interval is sensitive to the size of the standard error; large standard errors indicate that much of the cost variation in the observed data is unexplained by the equation" (Batchelder, et al, 1969:53). A 50% prediction interval is not very definitive; an interval of higher percentage would be meaningless.

Even with the wide intervals presented, one actual cost did not fall inside the interval. That one being the interval using Model I, Data Set I for the F-15. It should also be noted that the lower limits of the intervals for Model V, Data Set II were actually negative and were truncated to zero.

Looking closer at the interval widths, they range from 518.022 to 2387.824. Both extremes are relatively large, and the larger the interval width, the less certainty there is in the prediction.

As was done in the case of prediction errors, average intervals were computed for each model and each data set for the two airframes. These computations appear in Table XI.

TABLE X

50% PREDICTION INTERVALS

MODEL	DATA SET	ACFT	ACTUAL COST	50% PREDICTION INTERVAL	INTERVAL WIDTH
I	I	F-14	1050.833	(731.019,1452.281)	721.262
		F-15	1032.353	(505.553,1023.575)	518.022
II	I	F-14	1050.833	(863.924,1555.252)	691.328
		F-15	1032.353	(642.732,1172.576)	529.844
	II	F-14	1050.833	(59.251,2061.049)	2001.799
		F-15	1032.353	(77.678,1659.158)	1581.480
V	I	F-14	1050.833	(854.762,1463.046)	608.284
		F-15	1032.353	(570.920,1147.744)	576.824
	II	F-14	1050.833	(0 ,2387.824)	2387.824*
		F-15	1032.353	(0 ,1820.033)	1820.833*

All costs expressed in millions 75$.

*Actual interval width was wider than listed. Lower bound of prediction interval was truncated at zero.

TABLE XI

AVERAGE MODEL - DATA SET

PREDICTION INTERVALS

		DATA SET I	DATA SET II
MODEL	I	619.642	-
	II	610.586	1791.395
	V	592.554	2104.329

Table XI contradicts the results of Table VIII in that Data Set I is superior to Data Set II. No conclusive results can be drawn concerning the models.

The average interval width for each model, each data set, and each airframe was computed next. These averages were derived by taking the average of the interval widths across all the observations for each item of interest. These averages are listed in Table XII.

TABLE XII

AVERAGE INTERVAL WIDTH

MODEL		DATA SET		AIRFRAME	
I	619.642	I	607.594	F-14	1282.099
II	1201.113	II	1947.784	F-15	1005.241
V	1348.241				

The results of Table XII are somewhat the opposite of those of Table IX. Here Model I gives the smallest average prediction interval width; Data Set I is far superior to Data Set II; and the F-15 gives the smallest average prediction interval width. In terms of model, data set, and airframe, that combination which gives the most accurate prediction also has the most unexplained variance when it comes to prediction intervals. In terms of models alone, the model with the best prediction ability does not give the smallest prediction interval. The model with the smallest average prediction intervals gives the worst average point estimate of the total cost. These contradictory results, combined with the results of Chapter IV, make an accurate and unbiased evaluation of the models difficult and frustrating.

VI. SUMMARY

This thesis represents an effort to explain the future based on the results of the past. It enters into the uncertain world of cost prediction. Only time can truly validate a model and, unfortunately, by the time the model is validated it no longer has value as a predictor.

The historical development of parametric cost estimation models for aircraft airframes has relied on three variables: weight, speed, and quantity. Past efforts to explain airframe costs using other variables have been fruitless (Large, et al, 1976). This study used a microeconomic production function augmented with the learning hypothesis and traditional CER variables of weight and speed. By doing so, it takes into account the time flow of airframe production and, thus, provides a means to optimize the production process.

For the program manager, airframe weight and aircraft speed are fixed. Therefore, volume and time, as incorporated by the models in this study, are the only manageable variables in the cost equation. By using volume and time as decision variables, the manager gains greater control over total program cost. By varying volume and time within the model, he can determine the effect of program changes on total cost. This can be important in an acquisition process where change is the rule rather than the exception.

Conclusion

The results of this study are confusing and contradictory. They reflect the uncertainty that is inherent in all cost prediction. No model is entirely without fault; no method without uncertainty. In the

words of Voltaire, "Doubt is not a pleasant mental state, but certainty is a ridiculous one." With this in mind, an attempt will be made to arrive at a conclusion.

The analysis in Chapter IV seems to indicate that the simplest is best. The Womer model as developed in Chapter I was statistically better than the Levenson model (or restricted Womer model as developed in Chapter IV) in only one case. However, it must be remembered that the Womer undiscounted cost model could not be statistically compared to the Levenson model. Chapter IV did demonstrate the importance of data to the analysis. When the more carefully chosen data set (Data Set II) was used, results, particularly MSE, did improve. The data used in this study was anything but optimal and many concessions were made in its use.

The predictions in Chapter V were far from conclusive. The apparent accuracy of some predictions was offset by the inaccuracy of others. No model clearly distinguished itself.

It is, therefore, the conclusion of this study that the more complicated Womer model is no better than the less complex Levenson model. The best predictors of aircraft airframe cost still appear to be weight, speed, and quantity.

Recommendations

The conclusions of this thesis should in no way inhibit future study in the area of aircraft airframe cost prediction and analysis. The basis of the Womer model seems sound and reasonable. To use just three variables to estimate the future cost of something so complicated as an airframe seems absurd. It seems reasonable to assume that each airframe

would have its own learning curve and returns to scale. A future study might investigate this point and attempt to estimate costs on a less aggregate basis.

Also, as has been stated several times, the data used in this study could be improved upon. With better, more reliable data, the results could be entirely different. As a second recommendation, it is suggested that the study be reaccomplished using a different data base.

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APPENDIX A

CONTRACTOR LETTER

APPENDIX A

July 24, 1978

Fairchild Republic Company
Public Relations
Farmingdale, Long Island, New York 11735

Dear Sir:

As a student at the Air Force Institute of Technology, I am presently collecting data to be used in a master's thesis effort. The thesis involves an airframe cost estimation model in which production rate is a decision variable. In order to carry out my research, I need data on the planned (not actual) volume and production rate for various air frames.

To assure uniformity, I am requesting that the data be taken from the first production contract planning documents and be effective as of the date of the signing of the first production contract. Specifically, I need the planned number of airframes to be produced and the planned production schedule for those airframes broken down by month, if possible.

I am requesting the above data on the A-10, F-105 weapons systems.

Please send the data to my thesis advisor, Dr N. Keith Womer, and he will forward it to me. The address is:

Dr N. Keith Womer
AFIT/ENS
Wright-Patterson AFB, Ohio 45433

Thank you for your time and your consideration is very much appreciated.

Sincerely,

John A. Long
Capt, USAF

APPENDIX B

COMPLETE DATA SET

APPENDIX B

COMPLETE DATA SET

ACFT	COST	WEIGHT	SPEED	QUANT	VOLUME	TIME
A-4	11.537	5072.	578.	1.	23.	34.
A-4	78.156	5072.	578.	10.	23.	34.
A-4	105.899	5072.	578.	20.	23.	34.
A-4	11.537	5072.	578.	1.	19.	33.
A-4	78.156	5072.	578.	10.	19.	33.
A-4	11.537	5072.	578.	1.	501.	63.
A-4	78.156	5072.	578.	10.	501.	63.
A-4	105.899	5072.	578.	20.	501.	63.
A-4	240.313	5072.	578.	72.	501.	63.
A-4	365.553	5072.	578.	166.	501.	63.
A-4	472.124	5072.	578.	234.	501.	63.
A-5	230.126	22800.	1147.	11.	17.	32.
A-5	230.126	22800.	1147.	11.	193.	98.
A-5	358.977	22800.	1147.	25.	193.	98.
A-5	737.291	22800.	1147.	77.	193.	98.
A-5	920.231	22800.	1147.	97.	193.	98.
A-5	1060.516	22800.	1147.	120.	193.	98.
A-6	72.554	16400.	563.	5.	28.	42.
A-6	192.962	16400.	563.	17.	28.	42.
A-6	274.635	16400.	563.	28.	28.	42.
A-6	72.554	16400.	563.	5.	12.	42.
A-6	72.554	16400.	563.	5.	162.	91.
A-6	192.962	16400.	563.	17.	162.	91.
A-6	274.635	16400.	563.	28.	162.	91.
A-6	341.113	16400.	563.	40.	162.	91.
A-7	44.047	11521.	595.	3.	7.	33.
A-7	58.708	11521.	595.	7.	7.	33.
A-7	44.047	11621.	595.	3.	855.	93.
A-7	58.708	11621.	595.	7.	855.	93.
A-7	152.353	11521.	595.	42.	855.	93.
A-7	368.393	11621.	595.	199.	855.	93.
A-7	656.358	11621.	595.	395.	855.	93.
A-7	769.471	11521.	595.	400.	855.	93.
A-7	830.159	11621.	595.	407.	855.	93.
A-7	900.215	11621.	595.	419.	855.	93.
A-7	1203.278	11621.	595.	569.	855.	93.
A-7	1315.999	11621.	595.	626.	855.	93.
A-7	1550.324	11621.	595.	786.	855.	93.
F-4	238.517	17320.	1220.	7.	10.	52.
F-4	238.517	17320.	1220.	7.	25.	52.
F-4	424.733	17320.	1220.	23.	25.	52.
F-4	238.517	17320.	1220.	7.	23.	60.
F-4	424.733	17320.	1220.	23.	23.	60.
F-4	238.517	17320.	1220.	7.	86.	73.
F-4	424.733	17320.	1220.	23.	86.	73.
F-4	661.357	17320.	1220.	47.	86.	73.
F-104	9.233	7954.	1150.	2.	371.	63.
F-104	102.525	7954.	1150.	19.	371.	63.
F-104	550.939	7954.	1150.	228.	371.	63.
F-104	655.037	7954.	1150.	269.	371.	63.
F-104	703.393	7954.	1150.	290.	371.	63.

ACFT	COST	WEIGHT	SPEED	QUANT	VOLUME	TIME
F-104	9.233	7954.	1150.	2.	550.	71.
F-104	102.526	7954.	1150.	19.	650.	71.
F-104	550.939	7954.	1150.	228.	550.	71.
F-104	655.037	7954.	1150.	269.	550.	71.
F-104	703.393	7954.	1150.	290.	650.	71.
F-104	9.233	7954.	1150.	2.	244.	39.
F-104	102.525	7954.	1150.	19.	244.	39.
F-104	550.939	7954.	1150.	228.	244.	39.
F-104	9.233	7954.	1150.	2.	719.	64.
F-104	102.525	7954.	1150.	19.	719.	64.
F-104	550.939	7954.	1150.	228.	719.	64.
F-104	655.037	7954.	1150.	269.	719.	64.
F-104	703.393	7954.	1150.	290.	719.	64.
F-104	9.233	7954.	1150.	2.	1330.	70.
F-104	102.525	7954.	1150.	19.	1330.	70.
F-104	550.939	7954.	1150.	228.	1330.	70.
F-104	655.037	7954.	1150.	269.	1330.	70.
F-104	703.393	7954.	1150.	290.	1330.	70.
F-105	316.337	19439.	1195.	15.	256.	45.
F-105	855.640	19439.	1195.	80.	256.	45.
F-105	1136.477	19439.	1195.	108.	256.	45.
F-105	1423.570	19439.	1195.	176.	256.	45.
F-105	316.937	19439.	1195.	15.	76.	45.
F-105	316.997	19439.	1195.	15.	15.	50.
F-105	316.997	19439.	1195.	15.	1187.	104.
F-105	855.540	19439.	1195.	80.	1187.	104.
F-105	1136.477	19439.	1195.	108.	1187.	104.
F-105	1423.370	19439.	1195.	176.	1187.	104.
F-105	1829.777	19439.	1195.	315.	1187.	104.
F-105	2302.049	19439.	1195.	495.	1187.	104.
F-105	316.997	19439.	1195.	15.	963.	98.
F-105	855.540	19439.	1195.	80.	963.	98.
F-105	1136.477	19439.	1195.	108.	963.	98.
F-105	1423.370	19439.	1195.	176.	963.	98.
F-105	1829.777	19439.	1195.	315.	963.	98.
F-105	2302.049	19439.	1195.	495.	963.	98.
F-105	316.997	19439.	1195.	15.	1321.	104.
F-105	855.540	19439.	1195.	80.	1321.	104.
F-105	1136.477	19439.	1195.	108.	1321.	104.
F-105	1423.570	19439.	1195.	176.	1321.	104.
F-105	1829.777	19439.	1195.	315.	1321.	104.
F-105	2302.043	19439.	1195.	495.	1321.	104.
F-111	498.344	32458.	1262.	18.	1726.	118.
F-111	2165.342	32458.	1262.	159.	1726.	118.
F-111	2377.979	32458.	1262.	183.	1726.	118.
F-111	3169.330	32458.	1262.	279.	1726.	118.
F-111	3909.010	32458.	1262.	373.	1726.	118.
F-111	4330.179	32458.	1262.	431.	1726.	118.
F-111	498.344	32458.	1262.	18.	1174.	106.
F-111	2165.342	32458.	1262.	159.	1174.	106.
F-111	2377.979	32458.	1262.	183.	1174.	106.

ACFT	COST	WEIGHT	SPEED	QUANT	VOLUME	TIME
F-111	3169.330	32458.	1262.	279.	1174.	106.
F-111	3909.010	32458.	1262.	373.	1174.	106.
F-111	4330.178	32458.	1262.	431.	1174.	106.
T-38	31.445	5346.	750.	2.	368.	77.
T-38	68.949	5346.	750.	6.	368.	77.
T-38	116.498	5346.	750.	19.	368.	77.
T-38	235.817	5346.	750.	69.	368.	77.
T-38	398.175	5346.	750.	213.	368.	77.
T-38	536.743	5346.	750.	357.	368.	77.
T-38	31.465	5346.	750.	2.	808.	108.
T-38	68.949	5346.	750.	6.	808.	108.
T-38	116.498	5346.	750.	19.	808.	108.
T-38	235.317	5346.	750.	69.	808.	108.
T-38	398.175	5346.	750.	213.	808.	108.
T-38	536.743	5346.	750.	357.	808.	108.
T-38	31.445	5346.	750.	2.	793.	95.
T-38	68.349	5346.	750.	6.	793.	95.
T-38	116.498	5346.	750.	19.	793.	95.
T-38	235.817	5346.	750.	69.	793.	95.
T-38	398.175	5346.	750.	213.	793.	95.
T-38	536.743	5346.	750.	357.	793.	95.

Cost (C) - cumulative undiscounted production cost in million 75$

Weight (W) - AMPR weight in pounds

Speed (S) - maximum speed at best altitude in knots

Quant (Q) - cumulative quantity

Volume (V) - total planned production

Time (T) - total length of production period in months

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Source of Cost, Weight, Speed, Quantity:

Noah, J. Watson, et al. Aircraft Cost Handbook. Alexandria, Virginia: J. Watson Noah Associates, Inc., January 1971.

Source of Volume, Time:

Office of Assistant Secretary of Defense (Program Analysis and Evaluation), Acceptance Rates and Tooling Capacity for Selected Military Aircraft. Washington, DC: Department of Defense, October 1974.

APPENDIX C

DATA SET I

APPENDIX C

DATA SET I

ACFT	COST	WEIGHT	SPEED	QUANT	VOLUME	TIME	TIMEQ
A-4	11.537	5072.	578.	1.	520.	63.	23.
A-4	105.335	5072.	578.	20.	520.	63.	44.
A-4	78.156	5072.	578.	10.	520.	63.	38.
A-4	240.313	5072.	578.	72.	520.	63.	52.
A-4	365.555	5072.	578.	166.	520.	63.	57.
A-4	472.124	5072.	578.	234.	520.	63.	64.
A-5	230.125	22800.	1147.	11.	193.	98.	32.
A-5	358.377	22900.	1147.	25.	193.	98.	50.
A-5	737.291	22900.	1147.	77.	193.	98.	73.
A-5	920.291	22900.	1147.	97.	193.	98.	80.
A-5	1060.515	22800.	1147.	120.	193.	98.	90.
A-6	72.554	16400.	563.	5.	190.	91.	20.
A-6	192.552	15400.	563.	17.	190.	91.	40.
A-6	274.535	16400.	563.	28.	190.	91.	47.
A-6	341.119	15400.	563.	40.	190.	91.	54.
A-7	44.047	11621.	595.	3.	862.	93.	33.
A-7	58.708	11621.	595.	7.	862.	93.	37.
A-7	152.353	11621.	595.	42.	862.	93.	47.
A-7	368.395	11621.	595.	199.	852.	93.	58.
A-7	656.359	11521.	595.	395.	862.	93.	72.
A-7	769.471	11621.	595.	400.	852.	93.	72.
A-7	830.169	11621.	595.	407.	862.	93.	73.
A-7	900.216	11521.	595.	419.	862.	93.	77.
A-7	1203.279	11621.	595.	569.	862.	93.	85.
A-7	1315.399	11621.	595.	626.	862.	93.	88.
A-7	1550.324	11521.	595.	786.	862.	93.	95.
F-4	238.517	17320.	1220.	7.	86.	73.	54.
F-4	424.733	17320.	1220.	23.	86.	73.	66.
F-4	661.357	17320.	1220.	47.	86.	73.	77.
F-104	9.233	7954.	1150.	2.	1330.	70.	17.
F-104	102.525	7954.	1150.	19.	1330.	70.	28.
F-104	550.939	7954.	1150.	228.	1330.	70.	51.
F-104	655.037	7954.	1150.	269.	1330.	70.	55.
F-104	703.393	7954.	1150.	290.	1330.	70.	57.
F-105	316.337	19439.	1195.	15.	1187.	104.	69.
F-105	855.640	19439.	1195.	80.	1187.	104.	86.
F-105	1136.477	19439.	1195.	108.	1187.	104.	94.
F-105	1423.370	19439.	1195.	176.	1187.	104.	100.
F-105	1829.777	19439.	1195.	315.	1187.	104.	109.
F-105	2302.049	19439.	1195.	495.	1187.	104.	122.
F-111	498.944	32458.	1262.	18.	1174.	106.	48.
F-111	2165.372	32458.	1262.	159.	1174.	106.	83.
F-111	2377.379	32458.	1262.	183.	1174.	106.	85.
F-111	3169.330	32458.	1262.	279.	1174.	106.	98.
F-111	3909.010	32458.	1262.	373.	1174.	106.	110.
F-111	4330.178	32458.	1262.	431.	1174.	106.	116.
T-38	31.445	5346.	750.	2.	793.	95.	53.
T-38	68.349	5346.	750.	6.	793.	95.	61.
T-38	116.495	5346.	750.	19.	793.	95.	72.
T-38	235.917	5346.	750.	69.	793.	95.	82.
T-38	398.175	5346.	750.	213.	793.	95.	96.

ACFT	COST	WEIGHT	SPEED	QUANT	VOLUME	TIME	TIMEQ
T-38	536.743	5346.	750.	357.	793.	95.	98.

Cost (C) - cumulative undiscounted production cost in million 75$

Weight (W) - AMPR weight in pounds

Speed (S) - maximum speed at best altitude in knots

Quant (Q) - cumulative quantity

Volume (V) - total planned production

Time (T) - total length of production period in months

TimeQ (t) - time in months to accept quantity Q airframes

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Source of Cost, Weight, Speed, Quantity:

Noah, J. Watson, et al. Aircraft Cost Handbook. Alexandria, Virginia: J. Watson Noah Associates, Inc., January 1971.

Source of Volume, Time, TimeQ:

Office of Assistant Secretary of Defense (Program Analysis and Evaluation), Acceptance Rates and Tooling Capacity for Selected Military Aircraft. Washington, DC: Department of Defense, October 1974.

APPENDIX D

DATA SET II

APPENDIX D

DATA SET II

ACFT	COST	WEIGHT	SPEED	QUANT	VOLUME	TIME	TIMEQ
A-4	11.537	5072.	578.	1.	520.	63.	23.
A-4	78.136	5072.	578.	10.	520.	63.	38.
A-4	105.999	5072.	578.	20.	520.	63.	44.
A-5	230.126	22900.	1147.	11.	193.	98.	32.
A-5	358.377	22900.	1147.	25.	193.	98.	50.
A-5	737.291	22900.	1147.	77.	193.	98.	73.
A-5	920.231	22900.	1147.	97.	193.	98.	80.
A-5	1060.515	22900.	1147.	120.	193.	98.	90.
A-6	72.554	15400.	563.	5.	190.	91.	20.
A-6	192.552	15400.	563.	17.	190.	91.	40.
A-6	274.538	15400.	563.	28.	190.	91.	47.
A-6	341.119	15400.	563.	40.	190.	91.	54.
A-7	44.047	11621.	595.	3.	862.	93.	33.
A-7	58.708	11621.	595.	7.	862.	93.	37.
A-7	152.353	11621.	595.	42.	862.	93.	47.
A-7	368.395	11621.	595.	199.	862.	93.	58.
A-7	656.353	11621.	595.	395.	862.	93.	72.
A-7	769.471	11621.	595.	400.	862.	93.	72.
A-7	830.153	11621.	595.	407.	862.	93.	73.
A-7	900.216	11621.	595.	419.	862.	93.	77.
A-7	1203.279	11621.	595.	569.	862.	93.	85.
F-4	238.317	17320.	1220.	7.	86.	73.	54.
F-4	424.733	17320.	1220.	23.	86.	73.	66.
F-104	9.233	7954.	1150.	2.	1330.	70.	17.
F-104	102.325	7954.	1150.	19.	1330.	70.	28.
F-105	316.337	19439.	1195.	15.	1187.	104.	69.
F-111	498.344	32458.	1262.	18.	1174.	106.	48.
T-38	31.445	5346.	750.	2.	793.	95.	53.
T-38	68.349	5346.	750.	6.	793.	95.	61.
T-38	116.439	5346.	750.	19.	793.	95.	72.

Cost (C) - cumulative undiscounted production cost in million 75$

Weight (W) - AMPR weight in pounds

Speed (S) - maximum speed at best altitude in knots

Quant (Q) - cumulative quantity

Volume (V) - total planned production

Time (T) - total length of production period in months

TimeQ (t) - time in months to accept quantity Q airframes

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Source of Cost, Weight, Speed, Quantity:

Noah, J. Watson, et al. Aircraft Cost Handbook. Alexandria, Virginia: J. Watson Noah Associates, Inc., January 1971.

Source of Volume, Time, TimeQ:

Office of Assistant Secretary of Defense (Program Analysis and Evaluation), Acceptance Rates and Tooling Capacity for Selected Military Aircraft. Washington, DC: Department of Defense, October 1974.

APPENDIX E

F-14 AND F-15 DELIVERY SCHEDULES

~~CONFIDENTIAL~~ (When filled in)

U.S. NAVY F-14A TOMCAT

WEAPON SYSTEM PLANNING DATA
PROCUREMENT, INVENTORY, MILESTONES (AIRCRAFT) NAVAIR FORM 13010/1A (7-67)

AIRCRAFT MODEL DESIGNATION	POWER PLANT (Model and quantity)	ADMINISTRATIVE LEAD TIME	DEVELOPMENT LEAD TIME	DATE
VFX	TF30- P12 (2)	10	18	18 APRIL 1968

SECTION A - PROCUREMENT

FISCAL YEAR	NUMBER	STATUS OF BUDGET OR CONTRACT
& PRIOR		
69	3	(R&D Non Prog.)
70	9	(3 R&D Non Prog)
71	30	
72	96	
73	96	
74	96	
75	96	
76	43	

TOTAL ACCEPTANCES PRIOR TO (DATE) (NUMBER) AIRCRAFT

PRODUCTION SCHEDULE

CY	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
71	1	1	1	1	1	1	1	1	1	1	1	1
72	1	1	1	1	1	1	2	2	4	4	6	6
73	8	8	8	8	8	8	8	8	8	8	8	8
74	8	8	8	8	8	8	8	8	8	8	8	8
75	8	8	8	8	8	8	8	8	8	8	8	8
76	8	8	8	8	8	8	8	8	8	8	8	8
77	8	8	8	8	8	3						

SECTION B - INVENTORY

AIRCRAFT INVENTORY AND OPERATING NUMBERS (At end of time periods)

	CY 1971 MAR	JUN	SEP	DEC	CY 1972 MAR	JUN	SEP	DEC	CY 1973 JUN	DEC	CY 1974 JUN	DEC	CY 1975 JUN	DEC	CY 1976 JUN	DEC	CY 1977 JUN	DEC	CY 19 JUN	DEC	CY 19 JUN	DEC
* TOTAL INVENTORY (Non-Program & Program)		6	9	12	15	18	25	40	87	131	177	219	262	302	343	382	421	410				
LOGISTIC SUPPORT							2	8	27	41	38	48	43	49	54	56	76	65				
* TOTAL OPERATING (Non-Program & Program)		6	9	12	15	18	23	32	60	90	139	171	219	253	289	326	345	345				
NON-PROGRAM - TEST		6	9	11	13	14	14	6	6	6	6	6	6	6	6	6	6	6				
PROGRAM OPERATING					2	4	9	26	54	84	133	165	213	247	283	320	339	339				
NAVAIRLANT								17	33	51	69	87	94	96	96	118	137	137				
NAVAIRPAC						2	4	4	16	28	59	73	114	146	182	197	197	197				
NATRA																						
NATHESTRA																						
NARS																						
NATAL				1	2	2	5	5	5	5	5	5	5	5	5	5	5	5				

SECTION C - MILESTONES

SELECTED MILESTONE EVENTS	DATE (Mo & Yr)
NAVY PRELIMINARY EVALUATION (NPE)	APR 71
BOARD OF INSPECTION AND SURVEY (BIS)	JUN 72
NAVY SUPPORT	JAN 74
FLEET INTRODUCTION (First Aircraft)	
READINESS TRAINING SQUADRON NO. 1	NOV 72
READINESS TRAINING SQUADRON NO. 2	—
COMBAT SQUADRON NO. 1	APR 73
COMBAT SQUADRON NO. 2	JUN 73

REMARKS

* Includes 6 R&D Non Prog A/C

Classification Changed to UNCLASSIFIED
By Authority of E.O. 10501 Dated 11-5-53
Regraded by [signature] On OCT 10 1973
(Signature) (Date)

~~CONFIDENTIAL~~

McDonnell Douglas Corporation
P. O. Box 516
St. Louis, Mo. 63166

AIR FORCE F-15 BASELINE (1)

MO/YR	PRODUCTION A/C F / TF 75	F / TF 76	F / TF 77	F / TF 78	F / TF 79	TOTAL F-15: F-15 MO/CUM	TF-15 MO/CUM	ALL MO/CUM
Jul '76	11 1					11 121	1 18	12 139
Aug	10 2					10 131	2 20	12 151
Sep	10 2					10 141	2 22	12 163
Oct	11 1					11 152	1 23	12 175
Nov	10 2					10 162	2 25	12 187
Dec '76	10 2					10 172	2 27	12 199
Jan '77	11 1					11 183	1 28	12 211
Feb	10 2					10 193	2 30	12 223
Mar	10 2					10 203	2 32	12 235
Apr	11 1					11 214	1 33	12 247
May	10 2					10 224	2 35	12 259
Jun	10 2					10 234	2 37	12 27[illegible]
Jul	124 20	11 1				11 245	1 38	12 283
Aug	(FY75 - 144)	10 2				10 255	2 40	12 295
Sep		10 2				10 265	2 42	12 307
Oct		11 1				11 276	1 43	12 319
Nov		10 2				10 286	2 45	12 331
Dec '77		10 2				10 296	2 47	12 343
Jan '78		11 1				11 307	1 48	12 355
Feb		10 2				10 317	2 50	12 367
Mar		10 2				10 327	2 52	12 379
Apr		11 1				11 338	1 53	12 391
May		10 2				10 348	2 55	12 403
Jun		10 2				10 358	2 57	12 415
Jul	(MODEL TOTAL -	124 20)	11 1			11 369	1 58	12 427
Aug		(FY 76 - 144)	10 2			10 379	2 60	12 439
Sep			10 2			10 389	2 62	12 451
Oct			11 1			11 400	1 63	12 463
Nov			10 2			10 410	2 65	12 475
Dec '78			10 2			10 420	2 67	12 487
Jan '79			11 1			11 431	1 68	12 499
Feb			10 2			10 441	2 70	12 511
Mar			10 2			10 451	2 72	12 523
Apr			11 1			11 462	1 73	12 535
May			10 2			10 472	2 75	12 547
Jun			10 2			10 482	2 77	12 559
Jul		(MODEL TOTALS -	124 20)	11 1		11 493	1 78	12 571
Aug		(FY 77	- 144)	10 2		10 503	2 80	12 583
Sep				10 2		10 513	2 82	12 595
Oct				11 1		11 524	1 83	12 607
Nov				10 2		10 534	2 85	12 619
Dec '79				10 2		10 544	2 87	12 631
Jan '80				11 1		11 555	1 88	12 643
Feb				10 2		10 565	2 90	12 655
Mar				10 2		10 575	2 92	12 667
Apr				11 1		11 586	1 93	12 679
May				10 2		10 596	2 95	12 691
Jun				10 2		10 606	2 97	12 703
Jul			(MODEL TOTALS -	124 20)	10 2	10 616	2 99	12 715
Aug			(FY 78 -	144)	10 2	10 626	2 101	12 727
Sep					10 2	10 636	2 103	12 739
Oct '80					8 2	8 644	2 105	10 749
				(MODEL TOTALS -	38 8)			
				(FY 79 -	46)			

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NOTE: Original copy was of poor reproductive quality.

PROPOSED
F-15 DELIVERY SCHEDULE

Delivery Schedule No. 12.38
Date: 10 March 1971

MO/YR	RDT&E A/C (a) CATEGORY I F-15	CATEGORY I TF-15	PRODUCTION A/C CAT. II F-15	OPERATIONAL F	OPERATIONAL TF	OPERATIONAL F	OPERATIONAL TF	SERIAL A/C NO.	TOTAL F-15 MO	TOTAL F-15 CUM	TOTAL TF-15 MO	TOTAL TF-15 CUM	ALL MO	ALL CUM
	← CONTRACT NO. F33657-70-C-0300 →													
Jul '72	1							1	1	1			1	1
Aug	0							-	[illegible]	1			0	1
Sep	1							2	1	2			1	2
Oct	0							-	0	2			[illegible]	2
Nov	1							3	1	3			1	3
Dec '72	0							-	0	[illegible]			0	3
Jan '73	1							[illegible]	1	[illegible]			1	4
Feb	0							-	0	4			0	4
Mar	1							5	1	5			1	5
Apr	0							[illegible]	[illegible]	[illegible]			1	6
May	0	1						-	0	[illegible]	1	1	0	6
Jun	1	0						7	1	[illegible]	0	1	1	7
Jul	1	0						8	1	7	0	1	1	8
Aug	1	0						9	1	8	0	1	1	9
Sep	1	0						10	1	[illegible]	0	1	1	10
Oct	0	0						-	0	[illegible]	0	1	0	10
Nov	1	0						11	1	10	0	1	1	11
Dec '73	(12)	1 (2)						12	0	10	1	2	1	12
Jan '74	(FY 71 = 12)							-	0	10	0	2	0	12
Feb								-	0	10	0	2	0	12
Mar								13	1	11	0	2	1	13
Apr			1					14	1	[illegible]	[illegible]	2	1	14
May			1 (b)					15	1	[illegible]	0	2	1	15
Jun			1					16	[illegible]	[illegible]	[illegible]	2	1	16
Jul			1					17	1	[illegible]	0	2	1	17
Aug			1					18	1	[illegible]	0	2	1	18
Sep			1					19	1	17	0	2	1	19
Oct			1					20	1	[illegible]	0	2	1	20
Nov			1					22	[illegible]	[illegible]	1	[illegible]	[illegible]	21
Dec '74			(8) (FY 72 = 9)		1			(d) 24 & 21	[illegible]	[illegible]	1	[illegible]	[illegible]	23
Jan '75				1	1			(d) 23, [illegible]	2	21	1	[illegible]	[illegible]	25
Feb				2	1			[illegible], 27	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
Mar				2	1 (c)			30 - 32	[illegible]	[illegible]	0	[illegible]	3	32
Apr				3	0			33 - 35	[illegible]	27	1	7	3	35
May				[illegible]	1			36 - 38	3	31	0	7	3	38
Jun				3	0			39 - 41	[illegible]	[illegible]	1	[illegible]	3	41
Jul				3	1			42 - 44	[illegible]	[illegible]	0	[illegible]	3	44
Aug				3	0			45 - 47	[illegible]	38	1	[illegible]	3	47
Sep				2	1			48 - 50	3	41	[illegible]	[illegible]	3	50
Oct				3	0			51 - 55	[illegible]	[illegible]	1	10	5	55
Nov			(Model Totals (FY 73	(23)	(7) 30)	4	1	56 - 61	[illegible]	[illegible]	[illegible]	10	7	62
Dec '75						[illegible]	0	62 - 68	[illegible]	[illegible]	1	11	[illegible]	68
Jan '76						[illegible]	1	69 - 77	[illegible]	[illegible]	1	12	9	77
Feb						8	1	78 - 86	[illegible]	73	1	13	[illegible]	86
Mar						8	1	87 - 96	[illegible]	[illegible]	1	14	10	96
Apr						[illegible]	1	97 - 106	[illegible]	91	1	15	10	106
May						[illegible]	1	107 - 116	[illegible]	[illegible]	1	16	10	116
Jun '76						[illegible]	1	117 - 127	[illegible]	110	1	17	11	127
			(Model Totals FY 74			69	8 77)							

NOTES:
(a) First Flight Delivery in Month Shown.
(b) First Flight Provisional Acceptance in February 1974, prior to ECM Tests.
(c) First Flight Provisional Acceptance in December 1974, prior to ECM Tests.
(d) PCA on A/C No.'s 21 & 23 prior to delivery.

For deliveries beyond June '76 see Schedule No. 12.33, dated 29 Oct '[illegible]

RJO:D.13

VITA

John A. Long was born in Harrisburg, Pennsylvania on October 27, 1946. Spending most of his youth in Ohio, he graduated from Grove City (Ohio) High School in 1964. That same year, he entered Miami University, Oxford, Ohio, graduating in 1968 with an AB degree in mathematics. Upon graduation, he was commissioned a second lieutenant in the United States Air Force. Following graduation from undergraduate navigator training in 1969 and navigator-bombardier training in 1970, he served as a navigator and radar navigator in the Strategic Air Command. In 1976, he was awarded a Master of Arts degree in management and supervision from Central Michigan University. He entered the Air Force Institute of Technology in June, 1977.

He is married to the former Nancy Minot Peck of Cincinnati, Ohio. They have two daughters, Amy and Abigail.

Permanent Address: 2591 Bryan Circle
Grove City, Ohio 43123

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Nonlinear Least Squares Regression
Cost Prediction

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In recent years, men and governments have become keenly aware of the huge capital outlays necessary in the acquiring of new weapons systems. Increased burden on limited capital has required more complete and careful planning. This planning has led to the need for accurate and timely cost predictions of new systems.

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Historically, the variables affecting the future cost of aircraft airframes have been proven to be airframe weight and aircraft speed. These are often combined with a learning hypothesis to form an airframe cost model. In this paper, the production function of microeconomics is combined with weight, speed, and learning to form a nonlinear cost estimation model.

Nonlinear least squares regression analysis was used in evaluating this model. Although the results are inconclusive, based on the data used, weight and speed combined with learning still appear to be the best predictors of aircraft airframe cost.